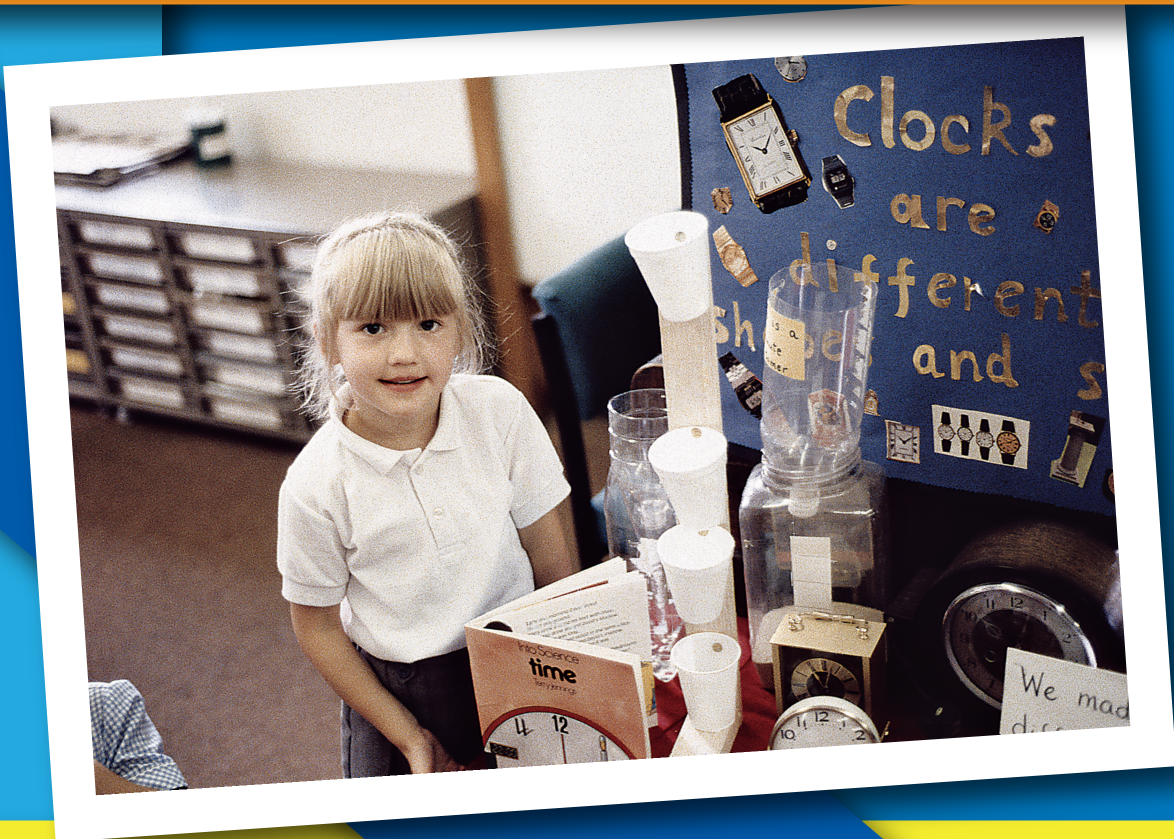
I DO, AND I UNDERSTAND

Helping Young Children Discover Science and Mathematics



Robert Louisell

with special guest chapters by

Stephen Hornstein and Peter Frost

I hear, and I forget.
I see, and I remember.
I do, and I understand.

*Ancient Asian Proverb.

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Author's Preface

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— Ancient Asian Proverb

There are many ways that young children learn. They learn from observing. They learn from actively listening and from overhearing. Most of all, they learn from doing. This book has been written for future teachers of young children; that is, students of early childhood education or child development. It has been specifically designed for use in courses on how to teach young children about mathematics and science.

A young child's thoughts are intricately tied to her or his actions because the young child's thoughts have not yet been fully distinguished from the young child's actions in the child's mind. Chapters 2 and 3 attempt to explain why this is so. Some education texts include a chapter on children's "misconceptions," as does this text, but, as far as we are aware, no text actually deals with appropriate strategies for interviewing children and interpreting what they say. Chapters 2 and 3 were written to help prospective teachers of young children learn how to better get to know the ideas of the children with whom they work. This is an essential skill for constructivist teachers since they must anticipate, and respond to, the ideas of their students.

But the whole point of this book—its theme—is that young children best learn about math and science by *do*ing. What does that mean? *Do*ing? It means that they learn how to add by combining groups of objects and counting how many objects there are in the new group that has been formed as a result. It means that they learn about density by making clay boats and testing them to see if they float. It means learning about electricity by being given a battery, a bulb, and a wire and figuring out how to make the bulb light. It means learning how to divide by taking a group of objects—say, caramels—and dividing them equally among themselves.

In other words, young children need to *experience* mathematics and science if they are to learn it well. And they need to *understand* these experiences. Children can learn the procedure for addition—combining two or more groups of objects and counting up the result—but they must also understand this process! When it comes to the written symbols for mathematics, the child must realize that two or more groups are being combined every time she sees a "+" sign.

A child's curiosity must be nurtured during instructional experiences with math and science. Telling children the answers to science questions and showing them the most efficient ways to solve math problems does not nurture their curiosity about these subjects. If you always answer the child's questions about science, what reason does she have to explore these phenomena further? If you always show the child how to solve math problems, when will she ever develop her *own* ideas about math? We don't mean that a teacher should never show a child how to do something like add or subtract or that a teacher should never tell the child an answer to a science question, but this sort of showing and telling should be done sparingly. Mathematics and science must also be related to the real life experiences of children. Otherwise, how will young children ever become aware of the importance of math and science in the world in which they live and observe things? Why will children want to learn about math and science? They must see how it applies to their everyday lives. Chapters 10 and 11 deal with this issue—how math and science are related to our everyday lives as well as how they relate to other subjects.

Our philosophy of teaching science and mathematics to young children is constructivist (See Chapter 1 for our interpretation of this term). Some people think that knowledge can be transmitted—*taught*—to children. We believe, as Piaget did, that the young child *invents* her or his knowledge and understandings. Teachers can facilitate this process of invention through a variety of methods; for example, by providing young children with hands-on experiences and engaging them in discussions and debates about these experiences. Children come to school curious about many things, including science and mathematics. It's a shame that they often leave school less curious about these things because we see the development of the child's curiosity as a primary goal—an essential standard—for school science and mathematics.

Although most textbooks of this type typically start with theory and move to examples of best teaching practices, the first three chapters of this book may have "more than enough" theory for some! Professors may choose to deemphasize these chapters, although I do hope that they will at least engage their students in the activity of doing clinical interviews with children. In my own experience as a college professor, it provides early childhood majors with a much needed insight into the differences between how children and adults think. As my doctoral advisor, Jack Easley, said to me many times, the most practical idea is to have a good theory.

We wrote this book because previous texts on the subject of early childhood mathematics and science teaching omitted an essential aspect of the field. While they covered preschool and kindergarten years, they neglected the mathematics and science teaching for grade levels one through three. This book corrects that fault. It comprehensively deals with preschool/kindergarten *and* grades 1–3 and it contains over a hundred tested early childhood activities for mathematics and science. The *Activities for Children,* which are numbered in each of the chapters that include them, are intended for teacher-candidates as well as children. It is appropriate to demonstrate these during class time or to have students practice these activities independently. It is assumed that the practicing teacher will make her own judgement about whether any individual activity is best completed as a teacher-led, supervised activity or as an independent activity for children in small groups. In most cases, this should be obvious from the description of the activity.

There are a variety of boxed items in this text which have been provided for the benefit of the reader. These include *Activities for Children, Activities for Future Teachers, and Assessments.* It is assumed that the reader of this text will keep a student journal related to the activities for future teachers in the science-related chapters. Those *Activities For Future Teachers* that are science-related may be carried out independently by the reader at home or in class with other students, depending on the instructor's preference. We have also included boxes for relevant content standards; for example, NCTM *Principles and Standards, Benchmarks for Science Literacy, National Science Education Standards,* and *Next Generation Science Standards.*

In the chapters about the development of the child's understandings of mathematics (Chapters 4–6), you will find boxed items entitled *Assessment Activity;* for example, *Assessment Activity 4.1.* These are not intended as summative assessments. Rather, they should be considered formative in nature; that is, they are snapshots of "where the child is" in his or her development for this specific math topic at this particular point in time. No inferences about the child's capabilities in mathematics are appropriate here. Rather, these assessments are provided to help teachers acquire some insight into the child's ideas about this topic at a particular point in time so that the teacher can better decide how to teach this child.

Our philosophy of assessment favors authentic assessments for formative purposes. In other words, we believe that assessments should occur in the context of ongoing experiences in which the children are engaged. Reports about the educational progress of individual children should communicate what the child has learned, is learning, and is about to learn. We oppose grades or marks that compare students to each other, except for very general developmental "landmarks" that can help a parent to understand the nature of a child's special needs. We have not included any of the diagnostic assessments used for purposes of special education because many early childhood education programs are now combined with special education and courses in this area can deal more appropriately with this topic. We have, however, included some adaptations for special needs students in science during kindergarten and the primary grades.

We have opted for specific assessments in the form of boxed items rather than for dealing with the entire topic of assessment in a separate chapter on the topic. Readers who would like to know more about strategies for assessing the student's learning during early childhood and elementary school can be referred to Chapter 7 of our previous book, *Developing a Teaching Style–2e,* by Louisell and Descamps (Waveland Press, 2001).

Many future early childhood teachers are unconfident about their own knowledge of science content. The *Activities for Future Teachers* in Chapters 7–9 provide the reader with experiences that will help them develop knowledge related to content that they must teach in the primary grades; for example, *organisms* and *life cycles.*

Appropriate *internet resources* are provided throughout the text in the context of the topics being discussed. A DVD accompanies this text. As this book goes to press, it contains some examples of interviews with children. We hope to include videotapes from classroom math and science lessons, along with an introductory presentation about constructivist teaching, with the second printing.

This book represents the ideas of the primary author about the theory and practice of teaching science and mathematics to young children. He has developed these ideas over a period of over 40 years while teaching children and future teachers. He has tested almost all of the *Activities for Children* with classrooms of young children. We hope you will find this book useful.

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References

Louisell, Robert, and Jorge Descamps (2001). *Developing a Teaching Style–2e.* Prospect Heights, IL: Waveland Press.

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Chapter 5

Teaching Young Children about Number Operations during Kindergarten and the Primary Grades

"Can you do addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?"
"I don't know," said Alice. "I lost count."
"She can't do Addition," the Red Queen interrupted.

from *Alice's Adventures In Wonderland*, by Lewis Carroll

There are many ways to teach any particular skill or concept. There is no "one right way." But there are specific things which must be dealt with when teaching young children about operations on numbers. When we say "operations on numbers" or "number operations," we're referring to things like addition, subtraction, multiplication, and division. We complete operations on numbers so that we don't "lose count" like Alice did in the quote at the beginning of this chapter. Some of the specific things that must be dealt with in number operations are the meanings of the plus and minus signs. What should you tell your students about the meanings of these symbols? In the following sections, we will share our particular approach to teaching young children about operations on numbers. Although many different methods may be successful, every method used should develop the child's insight and understanding about what is actually occurring when operations on numbers are performed. Children typically begin to experience actual instruction on the number operations in kindergarten or grade 1, although they may have experienced activities related to addition or subtraction during preschool.

Also, we should note that most children bring their own, intuitive ideas about mathematics with them to school. As Carpenter et al (1999) have said,

> ... children enter school with a great deal of informal or intuitive knowledge of mathematics that can serve as the basis for developing understanding of the mathematics of the primary school curriculum. Without formal or direct instruction on specific number facts, algorithms, or procedures, children can construct viable solutions to a variety of problems. (Carpenter et al, 1999, p. 4).

Learning to Complete Operations on Numbers

During this chapter, we will explain our methods for teaching young children to complete the four basic number operations: addition, subtraction, multiplication, and division. We will also share a number of games and activities that we have used to help children practice these operations.

Learning to Add

At the heart of the addition operation is the idea that two or more groups are being combined. The plus sign is a symbol to indicate this combination of groups. Thus, before one teaches a child how to add, one must first teach her about *groups*. Some math curricula may use the more correct mathematics term, “sets”, when talking about groups. [1]

Young children are used to referring to *things*. Thus we talk with them about *groups* of things; that is, groups of objects. The specific objects used are not important. We might use interlocking cubes, pennies, or acorns [2]

Activity for Children 5.1

Have the children arrange themselves at tables or on the classroom rug in such a way that they each have their own separate space. Tell them to set out a group (or “set) of four of the objects that are being used. Next, tell the children to make a group of three and place them in another part of the space where they are working (See Figure 5.1 for an illustration of this). Repeat this activity several times, varying the amounts of each group that you tell them to create. Continue this activity more briefly, from day to day, until the children appear confident about what they are doing.

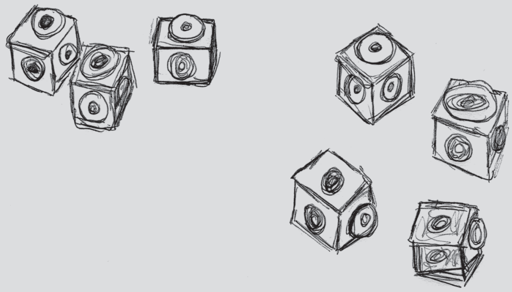

Figure 5.1

Activity for Children 5.2

Have the children arrange themselves where they have been working on making groups of objects. Begin the activity by having the children complete Activity for Children 5.1 (Have them make two separate groups at the spaces where they work). Next, tell them to “put together” the two groups of objects and “count them all up” and tell you how many of the objects they have in the new group—the one that they created by combining the two groups that they had made (See Figure 5.2). Repeat this activity often, from day to day, until the children appear confident about what they are doing.

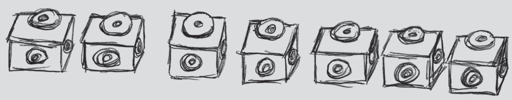

Figure 5.2

It's important that teachers of young children be aware that they should *not* introduce the symbols for the addition process when they first introduce the concept of addition. Symbols represent a more abstract level of thinking and children should first become familiar with doing the actual combining of sets before they are exposed to the written representation of addition. However, at some point, children will need to be introduced to the written symbols that represent what they are doing when they combine two groups and count up the result. Typically, their math workbooks will represent this activity as an equation in horizontal form. For example,

$$4 + 3 = 7$$

We have typically introduced these symbols in the following way.

Activity for Children 5.3

The teacher uses a board or chart paper to direct this activity. Writing the numeral 4 on the board, the teacher says "Make a group of this many" and waits for each of the children to create their groups of 4. Next, the teacher says "Now, make a group of this many" as she writes a 3 on the board. Next the teacher inserts a + symbol between the two numerals on the board. The written symbols look like this at this point:

$$4 + 3$$

Pointing to the plus sign, the teacher then says "This is called the plus sign. It means put them together. Go ahead! Put the two groups together. How many do you have altogether in your new group that you just made when you put them all together?" (Students should answer "7").

Next, the teacher introduces the "equals" sign by putting it just to the right of the 3. Thus, the written symbols should look like this:

$$4 + 3 =$$

The teacher says "This is called the 'equals' sign. It means 'count them up and tell me how many.'[3] How many did you say you have altogether? Seven? Okay." The teacher should then write the 7 to complete the number sentence. At this point, the equation should look like this.

$$4 + 3 = 7$$

Repeat this activity often, from day to day, until you think the children are reasonably comfortable with it. After this, the last step is to have them represent the addition operation with their own writing, on paper.

Types of Addition Problems

These three activities (5.1, 5.2, and 5.3) introduce the concepts of sets and of addition as combining two or more sets. However, the addition problems which a child may be faced with are actually of three types: 1. Those for which the result is not yet known (We call this type of addition problem a "Result Unknown" problem. An example of a "result unknown" problem is the type of problem completed in Activity for Children 5.3). 2. Those for which the amount to be added to the beginning is unknown (This type of addition problem can be called "Change Unknown"), and 3. Those for which the start is unknown (We can call this type of problem "Start Unknown") (Carpenter et al, p. 8).

These three types of addition problems are represented and illustrated in Figure 5.3.

Results Unknown Problems: Addition is always a combination—a joining—of one or more groups, but children often *think* of these problems in terms of actions required. For example, two groups must be combined for the "result unknown" type.

Change Unknown Problems: In contrast, the child must start with a group of a certain amount for the "Change Unknown" type of problem and then figure out how many more items must be added to that initial group in order to arrive at the result.

Start Unknown Problems: For the "Start Unknown" type of problem, the child must figure out what beginning amount can be combined with the addend that is known in order to end up with the stated result. So, although

Unknown	Example
Result Unknown	*Robin had 6 toy cars. Her parents gave her 8 more toy cars for her birthday. How many toy cars did she have then?*
Change Unknown	*Robin had 6 toy cars. Her parents gave her some more toy cars for her birthday. Then she had 14 toy cars. How many toy cars did Robin's parents give her for her birthday?*
Start Unknown	*Robin had some toy cars. Her parents gave her 8 more toy cars for her birthday. Then she had 14 toy cars. How many toy cars did Robin have before her birthday?*

Figure 5.3

Join Problem Types

Used with permission from *Children's Mathematics, 2nd Edition: Cognitively Guided Instruction* by Thomas P. Carpenter, et al.

each of these three types of problems is founded on the idea of combining (joining) groups, the order for which the child thinks of the separate groups and the result may vary. Because of this, children need many experiences with each type of problem if they are to become successful in addition of all types.

Activity for Future Teachers 5.1

Think of situations which can be expressed in the Result Unknown format and devise problems in that format. Do the same for the Change Unknown and Start Unknown formats. You may wish to share the problems you have devised as a class.

Learning to Subtract

Like learning to add, subtraction can be modeled with manipulatives and actions.

Subtraction As "Take-Away": The first, and easiest, type of subtraction problem that children can learn are problems of the separation or "take away" type.

Activity for Children 5.4

Have each child make a group of 8 objects. Now, tell them to "take away" 3 objects (Or, tell them to use objects in their group of 8 to make a group of 3 and then tell them to "take away" the group of 3 from the larger group). Have them tell you how many are left in the original group now that 3 objects have been removed (5). Repeat this activity many times, and on subsequent days, before introducing the symbols for the subtraction operation.

We noted for addition that teachers of young children should not introduce the written representation of the addition process until the children have become familiar with modeling of the addition process of combining groups. This is also true of subtraction. After children have become competent (and confident) at modeling the subtraction as a process of separating ("taking away") a set from a larger set, they should be introduced to the written representation of this type of subtraction.

Activity for Children 5.5

Write the numeral 7 on a board and tell the children to "Make a group of this many objects." Now put a minus sign just to the right of the 7, like this:

7 –

and insert a 5 next to the minus sign, like this:

7 – 5

Tell the children that the minus sign means to "take away" an amount. Tell them to take away five from the seven objects that they have set out.

Now, introduce the equals sign in this context. Say "Do you remember the equals sign from our addition problems? Subtraction problems use an equals sign, too." Write the equals sign next to the 5, like this:

7 – 5 =

Say "Count up how many are left now that you took away five from the seven that you had. How many are left?"[4]

Repeat this activity several times, on subsequent days, using different amounts. Keep the amount in the original group to 9 or less for now.

Part-Part-Whole Type of Problem: There are other types of subtraction than the simple, take-away type. For example, there is the type which ask for one of the sets that make up the whole. To illustrate this, let's examine the following word problem:

> *Bobby had 7 Magic Cards but then he gave some to Marie. After that, he only had 4 Magic Cards left. How many did he give to Marie?*

For the problem above, seven represents the "whole"—the amount that Bobby started with. Four represents one of the two sets, or "parts," that made up the whole. The other set (the other part) isn't specified. This type of problem is usually called a "Part-Part-Whole" type of problem

Comparison Problems: Some problems ask children to make comparisons. This type of problem does not refer to "parts" and "wholes." Rather, it asks the child to compare two distinct groups (sets) and to state the difference between their two amounts. For example, let's examine the following problem:

> *Rebecca has 5 pet fish. John has 7. How many more does John have?*

In the problem above, children are being asked to compare amounts—the amount of fish Rebecca has to the amount that John has—and to state the difference (2).

Why Types of Addition and Subtraction Problems Are Important

Many teachers don't bother with learning about—or exposing their children to—the various problem types. However, children must become familiar with the different types of addition and subtraction problems, even if they don't learn the names for them, because real-life experiences don't come packaged in "plus" and "minus" number sentences. Also, the standardized tests in mathematics that compare the performance of students in different countries to each other typically show that American children do as well as children in other countries at making calculations when given number sentences. However, they typically perform poorly on the sections of these tests that require children to solve word problems (Payne, 1988). As Carpenter et al put it (1999),

> Children's ability to solve word problems depends to a great degree on their ability to recognize the distinctions among the problem types ... Variations in the wording of the problems and the situations they depict can make a problem more or less difficult for children to solve (p. 10).

Activity for Future Teachers 5.2

Try to devise authentic word problems that exemplify each type of subtraction. You may wish to share your problems as a class.

About Addition and Subtraction Facts

Most mathematics educators recommend a particular order for learning about addition and subtraction. Young children need to understand what addition and subtraction are, and how to complete addition and subtraction problems,, before they should be asked to memorize addition and subtraction facts. Some people question whether children (or adults) need to know addition and subtraction facts in this era when calculators are provided on everyone's cell phone. But one does need to know facts in order to judge whether or not a calculator has made a mistake; that is, knowing arithmetic facts helps people to judge whether the result provided by a calculator is reasonable. Calculators can make mistakes for a variety of reasons; for example, low power. The most common cause of calculator mistakes is probably incorrect input of data; for example, entering the wrong number to be added.

Once young children have a secure knowledge of addition and subtraction, there are a variety of ways to go about helping children to commit math facts to memory. Before we discuss games that are useful for this purpose, we should first discuss some of the content knowledge related to addition facts that will be helpful regardless of how it is taught. The first and most helpful thing teachers can do to help children with remembering math facts is to help them understand the following addition patterns.

Doubles: Doubles turn out to be much easier to learn than most addition facts. In fact, children in first grade remembered 5 + 5 just as well as 2 + 2 in a study conducted by Kamii and others (Kamii, 2000, p. 74–76). Young children should be taught all doubles up to 10 + 10, once they understand the addition process. This is because they are easy to remember. Knowledge of doubles are also helpful because, if you know doubles, you can quickly figure out facts which differ from doubles by only one. For example, if you know that 4 + 4 = 8, you can quickly figure out that 4 + 5 should be one *more* than 8, or 9, and that 4 + 3 will be one *less* than 8, or 7 (Kamii, 2000, p. 77).

Plus 1: Adding one to any number is relatively easy to remember because the result is only *one more* than the number being added to; for example, 6 + 1 is 7, 3 + 1 is 4, etc. (Kamii, 2000, p. 76).

Plus 2: According to research conducted by Kamii and others, adding two to any number is also easier to remember than most addition facts (Kamii, 2000, p. 74–76).

Commutative Principle: When children develop *reversible* thought (about age 7–8—See Chapter 3 for more on this), they are more likely to figure out the commutative principle; that is, that 4 + 5 yields the same answer as 5 + 4 (and 3 + 5 = 5 + 3, etc.). However, some children never seem to notice this. For children who don't figure out the commutative nature of addition, you will need to teach them about it, by explaining that every problem with only two addends can be added, either addend to the other, and yield the same result. You should provide as many examples of this as needed to help the children to understand this (Kamii, p. 74–77).

Card Games for Practice With Addition

Card games are engaging and useful for helping children to discover and commit arithmetic facts to memory. They help children to develop social skills while, in comparison, children working individually at computers will not have much opportunity to do so. As a general rule, no more than 4 children should play a game together because it reduces the amount of time that each student gets for his or her own turn. The following games may be used to help children learn addition facts.

One More: For this game, children need a die with numerals 1–6 on it, four sets of transparent chips of different colors (10 of each color), and a game board with the numerals 2–7 on it, like this:

5	2	6	3	4	7	3
3	7	5	4	6	2	5
4	7	2	5	3	7	6
6	5	6	7	6	4	3
2	7	3	6	2	5	4

Figure 5.4
Game Board for One More

Children take turns rolling the die and placing one of their chips on a number that is one more than the number they have rolled with the die. This helps children to learn their "plus 1" facts (Kamii, p. 170–171).

Double War: This game is for two players only. To prepare this game, take all the 1, 2, 3, and 4 cards from two decks of cards. Shuffle them and set out 4 equal stacks of cards, two for each player. Stacks are placed face down. Children simultaneously take the top cards from each of their two stacks and turn them over. They add the numbers on each of their two cards together. The player with the highest total gets to take all four cards. An advantage of this game is that both children supervise each others additions because they need to know who has the highest total (Kamii, p. 171–172).

Cover-Up: For this game, use dice that are numbered from 1–6. You will also need 22 poker chips. Children sit on opposite sides of the game board (See Figure 5.5). Each child uses only his or her side of the board. The children take turns rolling two dice. After rolling, they add the two numbers that appear and cover up the number representing the total on their side of the board. If a total that has been rolled has been previously covered, that child's turn is wasted. The first child to cover up each number on his or her side of the board is the winner (Kamii, p. 173).

12	11	10	9	8	7	6	5	4	3	2
2	3	4	5	6	7	8	9	10	11	12

Figure 5.5

Double Parcheesi: This game is played like the commercially available game, Parcheesi, except for one thing: When children roll the die, they double the number that appears on the face of the die. Children like playing Double Parcheesi because it enables them to move their markers twice as fast along the long path on the board. Research has shown that doubles are easy to remember for young children (Kamii,, p. 74–77). As we have already mentioned, knowing doubles is useful for children because it helps them to figure out other totals besides doubles. For example, if children are asked to add 4+5, they can double the 4 and add 1. Or, they can double the 5 and subtract 1 (Kamii, p. 175).

Card Games for Practice With Subtraction

Any number (amount) above 1 can be partitioned. For example, we can think of 5 as five individual items (1 + 1 + 1 + 1 + 1) or we can think of it as 4 + 1 or as 3 + 2. The ability to partition numbers is the foundation for understanding subtraction. If a child knows that 3 +2 = 5, she will more easily understand and remember that 5 – 2 = 3 (and 5 –3 = 2). The following are games to help children understand and commit to memory subtraction facts.

Making 5 (or 6, or 7): [5] This game is best for two players. Players take turns and no "double turns" are allowed. Using two decks of cards select all cards with 1 to 4 on them (Aces are 1s) and place them together in one stack. Shuffle the stack. Deal 5 cards to each child. Take one card from the stack and place it face up on the playing surface (You will have one stack and one discard pile that shows the top card, face up). Player #1 draws a card and tries to combine one card from his or her hand with the number of the card in the discard pile to make 5. For example, if the number that is facing up is a 2, Player #1 must use a 3 card to make 5. If the child cannot make 5, she must select a card to discard and put it on top of the discard pile (facing up) and pass her turn to Player #2. If Player #1 *can* make 5, she takes both cards (the card on top of the discard pile and the card she combines with it to make 5) and places them together on her side of the playing surface so they can be totalled at the end of the game. Play continues until the cards are exhausted. The player with the highest total wins. This game may be adapted to "Making 6" or "Making 7" (or higher numbers) whenever the children are ready for it.

"Make Ten": Arrange the playing cards as pictured below (see Figure 5.6). Remove all the face cards from a normal deck of cards, leaving only the numbered cards (1–9). Player #1 finds all the pairs of cards in this arrangement that will add to 10. In this case, pairs that add to 10 are the two 5s, the 3 with the 7, and the 1 with the 9. Player #1 places her pairs on her side of the playing surface so they can be totalled at the end of the game. Player #1 then replenishes the empty spaces with cards from the stack of those cards that were not used to make the original

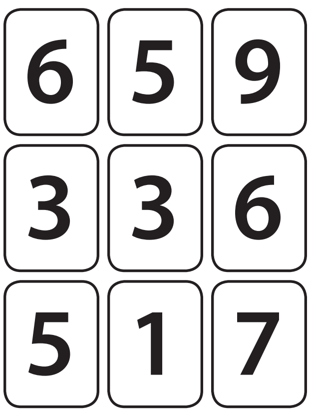

Figure 5.6
Game Board Arrangement To Begin "Make 10"

playing surface. Player #2 continues in the same fashion as Player #1. The player with the highest total when the entire deck of cards has been exhausted is the winner (Kamii, p. 182).

There are literally hundreds of games that teachers can adapt to help children to commit addition and subtraction facts to memory. Consult *Young Children Reinvent Arithmetic* by Constance Kamii (1985, 2000) for more.

Learning to Multiply

Do you remember our discussion of class inclusion in Chapter 4? It is much easier for a child to understand multiplication if they have already developed an understanding of class inclusion. And it is also much more difficult for a child to understand multiplication if the child has *not* developed class inclusion yet. We recommend that teachers conduct a quick assessment for class inclusion (such as the activity using A-Blocks which we described in Chapter 4) if they're uncertain about the child's understanding of class inclusion.

The first idea to emphasize when introducing multiplication to young children is that multiplication is *repeated addition*; that is, it is addition repeated several times. For example, if we lay out 3 rows of checkers with 5 checkers in each row), as in Figure 5.7, we can see that we have 3 rows with 5 checkers in each row. This can be represented in writing as $5 + 5 + 5$.

However, this same arrangement of checkers can be thought of as five *columns* with 3 checkers in each column. This can be represented as $3 + 3 + 3 + 3 + 3$. Either way, the total number of checkers is 15.

When introducing multiplication to children, we like to tell them that the symbol $\times$, in 5×3, means "5 added up 3 times." [6] The numbers in the equation actually represent different types of things. One number (3, if we think of 5 added up 3 times) represents how many groups there are (3) while the other number (5) represents how many items are in each group (Carpenter, et al, p. 34).

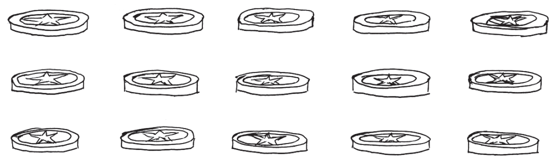

Figure 5.7

3 rows of checkers, 5 checkers in each row

Activity for Children 5.6

Tell the children to make 3 groups with 5 buttons in each group (as in Figure 5.7, laid out in an array with 3 rows of 5). Assist them as needed. Next, ask them how many groups of five they have set out. After they reply correctly (3), ask them how many buttons there are in each group (5). Next, ask them how many buttons they have altogether (15). Tell them "That's 3 groups of 5. Or, we could say that we added 5 up 3 times and we got 15. 5 + 5 + 5 equals 15 altogether!" Show the children how to consider the array from a different perspective, as 5 groups with 3 in each group. Repeat this procedure with several single-digit multiplication equations.

Activity for Children 5.7

To teach children about skip-counting, we like to use a number track that can be used with unifix cubes.[7] We begin by having the children align the first cube with the beginning of the track on the left-hand side. Next, we tell the children to use only two colors (for example, blue and white) and to lay a white cube down first and a blue cube next, and then to repeat this pattern (white-blue, white-blue) until they get to 20 (See Photo 5.1 below). Next, we tell the children to write down on paper all of the numbers that the blue cubes are underneath (for example, 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20). We tell them that this is "counting by twos." From that day on, we have the children pick up and put away the cubes in 2s whenever we are working on counting by 2s.

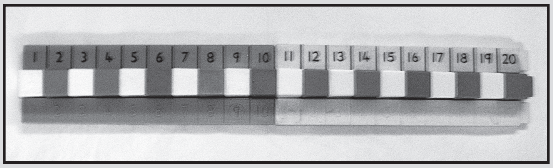

Photo 5.1

White-blue Sequence On Number Track (to 20)

Most children are already familiar with skip-counting by the time they are introduced to multiplication. Skip-counting is essentially repeated addition (Carpenter et al, p. 40). However, some children are only able to skip-count for a few times and must finish their calculation by counting individual objects by ones (Carpenter, et al, p. 40). For example, examine the following word problem.

> *The teacher has 5 sheets of stickers. There are 4 stickers on each sheet. How many stickers does the teacher have?*
>
> Jennie counts, "Let's see, 4, 8, 12 [pause] 13, 14, 15, 16 [pause] 17, 18, 19, 20—20 stickers." (Carpenter et al, p. 40).

You can devise similar approaches to counting by 3, 4s, 5s, and 10s. Since the number track goes all the way up to 100, we eventually have the children counting by 2s or 3s, etc., to 100 and putting the cubes away by 2s, 3s, etc. at the end of their math time for the day.

Activity for Children 5.8

Multiplication Game (Using Dominos):
The following game may be used to introduce children to the concept of multiplication and also to provide children with practice at multiplying. It requires a set of dominos. Dominos are ideal for introducing children to the concept of multiplication because each domino piece includes a specific number of dots on it. We recommend that you begin with a dominos set that does not contain amounts of more than 6 on any one domino piece.

(continued)

As with any instructional game for children, no more than 5 persons should be in one group playing the game. This assures that each child will get a turn without waiting too long. To introduce the game, lay a domino piece on the playing board.[8] Let's say you lay a piece with five dots on it, which would look like this (See Photo 5.2):

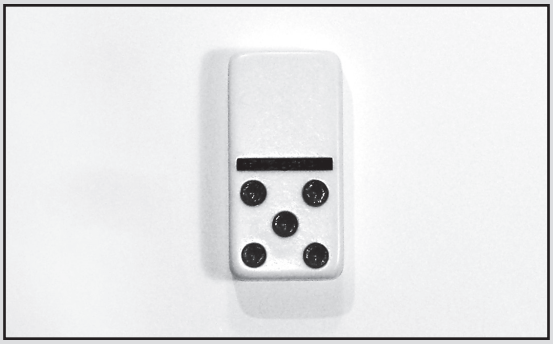

Photo 5.2

First Move of Dominos Multiplication Game

Say "I have one 'five.' That's five points for me." Have a child write "5" beneath your name on a piece of paper to represent your current score. Next, tell the children that play moves from "left-to-right" like a clock. Ask if the child seated to your left has a piece that matches yours. If not, continue to move left (around the clock) until you find a child who has a matching piece. When a child has a piece that matches yours, take the piece and lay it next to yours, like this (See Photo 5.3):

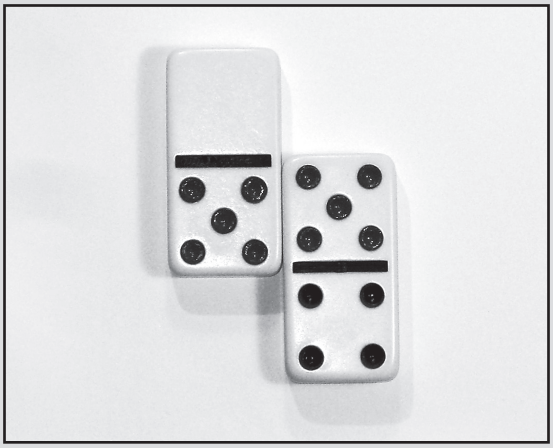

Photo 5.3

Second Move of Dominos Multiplication Game

Say "Two fives is 10—See!" and count the dots individually on each domino piece, counting from 1 to 10. "Two fives equals 10." Say "That's 10 points for you" and have a child write "10" next to that student's name. Continue play by matching pieces in the following way (See Photo 5.4):

Photo 5.4

Third Move of Dominos Multiplication Game

Say "Oh! You have two fours! That's four added up two times. That's eight" and have a child write "8" under that child's name on the piece of paper.

Play continues until time is called for completing the game. The player with the highest total wins.

This game is good when teachers are introducing the concept of multiplication because it helps children understand that multiplication is repeated addition. It is also a good game for providing practice with multiplication facts after the children have become comfortable with he concept of multiplication.

Teaching Children About Commutative and Identity Principles To Help Them Remember Multiplication Facts: Just as addition is commutative (You can combine the addends in any order and get the same result, for example 3 + 2 = 2 + 3, or 5), multiplication is also commutative (5 × 3 = 3 × 5). Some children figure this out on their own. Others need to have it pointed out for them. If a child realizes that multiplication is commutative, the number of multiplication facts that she needs to remember become half of what they would be if she did not realize this.

There is also an identity principle for addition and subtraction. For addition, it is zero. That is, zero added to any number yields that number; for example, 7 + 0 = 7.

Activity for Future Teachers 5.3

What number is the identity number for multiplication? In other words,when you multiply, what number can you combine with any number and get that same number as a result?

Teaching Children About Place-Value

At this point, your instructor for the course may be wondering why we put our discussion of place-value at this location in the chapter; that is, why did we include our discussion of place-value *after* multiplication. After all, place-value is introduced to children in most school curricula during the 1st or 2nd grade. However, place-value *is* multiplication. Take the number 26. It is comprised of 2 tens and 6 more. Another way to say this is 26 equals 2 times 10 plus 6. In this section, we will share some of the most effective approaches to teaching children about place-value. Bear in mind throughout our discussion, however, that one of the reasons so many young children have difficulty with this idea is that most young children have not yet developed a good understanding of class inclusion. Mathematics curricula in the primary grades don't emphasize multiplication until the child is likely to have developed an understanding of class inclusion, yet they include place-value in the second grade curriculum and introduce it in first grade, even though the concept of place-value is actually based on the same idea as multiplication.

Foundation For Learning About Place-Value: There are several things that teachers can do to prepare children to understand the concept of place-value. We have already mentioned skip-counting and how important it is that the child has developed a concept of class inclusion. Skip-counting helps children develop an insight that numbers may be grouped in many ways. For example, they may be grouped by twos or threes (or fours or fives), etc.

In addition to class inclusion and skip-counting, other numbers systems can help a child develop the insight that numbers may be organized in a variety of ways. For example, the Romans organized their numbers around a base of 5; that is, the numbers either added to or subtracted from 5. The Roman Numeral for 5 was V. 6 was VI (5+1) while 4 was IV (one less than 5). The Babylonians based their number system on groups of 10. Individual teeth were laid out facing down—one tooth for an amount of one, two for an amount of two, etc. When they wanted to show an amount of 10, they faced the tooth sideways instead of down (See Figure 5.8). For the Mayans, an amount of one was represented by

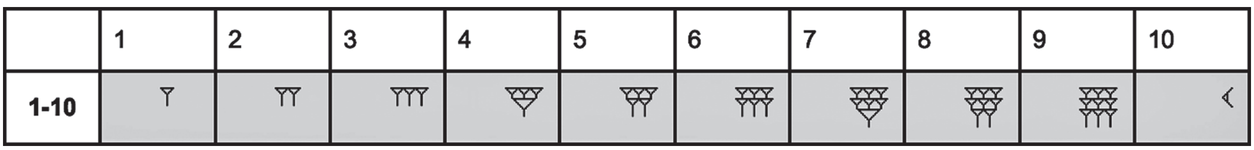

Figure 5.8

Babylonian Numbers, 1–10. *Source:* http://gwydir.demon.co.uk/jo/numbers/babylon/

a single stone (or dot, if written). Two was written as two dots. But a 5 was written like a horizontal stick (See Figure 5.9 for a depiction of the Mayan system). We like to spend at least a day on each of several number systems with a class of children in grade 2, having children explore these different number systems. We believe it helps children to understand that numbers can be grouped in many ways. The better a child understands this, the more likely the child is to understand how we group numbers within our own, base-10, number system.

Of course, children with special learning needs might be more confused than helped by learning another number system, but most children will gain further insight into how numbers may be organized in a variety of ways. This insight is important when children are about to be taught about our own system of organizing numbers in groups of 10.

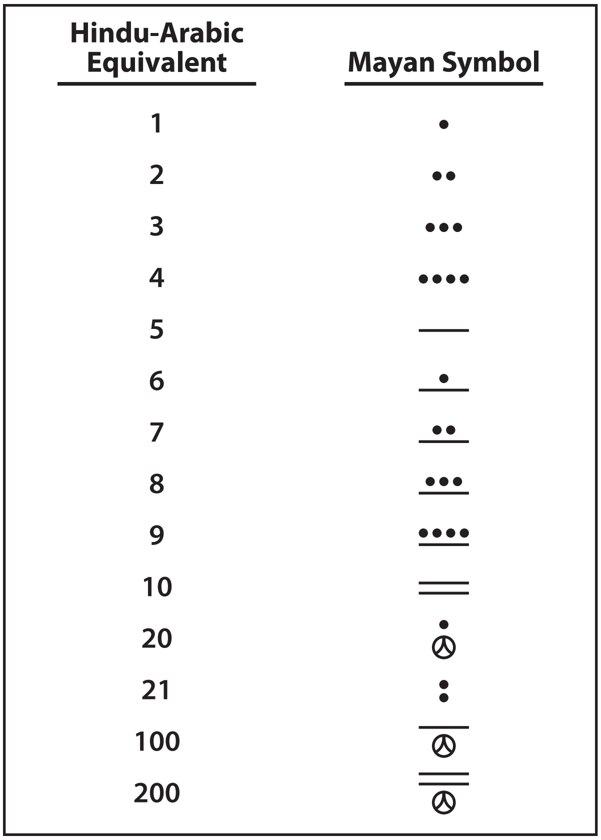

Figure 5.9

Mayan Numeral Equivalents

House Numbers. Another thing a teacher can do to prepare young children for learning about place-value is to take them on a walk through a few blocks in the neighborhood of the school, pointing out different house numbers and asking them to figure out how the addresses change from one house to the next and, especially from block to block. This activity is especially effective in cities, where the 4-digit house numbers change from, for example, 15xx to 16xx.

The Trading Game. If children are going to understand place-value, they will need to realize that they can exchange 10 ones for a ten and vice-versa. We have used the following game to help them develop this understanding.

Activity for Children 5.9

"Trading Game":
Children must create a "bank" containing individual base ten blocks and "tens" (See Photo 5.5).

Children take turns rolling a die (with 1–6 dots or numerals on each face of the die). The "banker" gives each child whatever amount they roll on the die. As soon as any child acquires ten or more blocks, they must trade them into the bank for a ten. The first child to acquire 5 tens is the winner. Children should play this game repeatedly from day to day until trading ones for tens and tens into ones becomes second nature to them. Note: Other manipulatives may be used for this game; for example, popsicle sticks that have been bundled in tens (or left as individuals) and unifix cubes that have been interlocked in sticks of tens (or left as individuals).

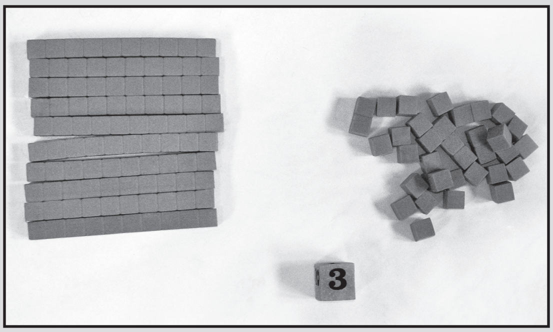

Photo 5.5

A Bank of 10s and 1s, with Die[9]

Teaching Children About Addition and Subtraction With Place-Value

During the primary grades, children are introduced to addition and subtraction of numbers with place-value; for example, the number 26 contains two tens and 6 ones. In this section, we explain our approach to teaching young children these concepts and skills.

Teaching Children How to Add With Two-Digit Numbers: Eventually, children need to learn how to add two-digit numbers. This skill is typically introduced in first or second grade. It is usually recommended that children be introduced to two-digit addition problems that don't require trading (This used to be called "carrying"). Thus, problems such as these may be presented to the children:

$$\begin{array}{r} 25 \\ +\ 32 \\ \hline \end{array} \qquad \begin{array}{r} 36 \\ +\ 41 \\ \hline \end{array} \qquad \begin{array}{r} 63 \\ +\ 21 \\ \hline \end{array}$$

Notice that, for these problems, numbers in the ones column do not add to 10. That would require trading (which used to be called "carrying"). However, the following problems would not be introduced yet, since the numbers in the ones column add to 10 or more:

$$\begin{array}{r} 25 \\ +\ 37 \\ \hline \end{array} \qquad \begin{array}{r} 36 \\ +\ 45 \\ \hline \end{array} \qquad \begin{array}{r} 63 \\ +\ 27 \\ \hline \end{array}$$

Activity for Children 5.10

To introduce two-digit addition problems that do not require trading ("carrying"), we like to provide each child with a simple chart, like this:

Tens	Ones

It's best to introduce concepts and skills without requiring the child to write anything. This is so the child will become familiar with the process of thinking through the actions involved in solving these problems (Recall our recommendation that all concepts be introduced in the enactive mode). Later, the written representation can be taught.

Provide each child with a t-chart ("Tens and Ones" chart) and a "bank" of unifix cubes.[10] Next, tell the children to take 25 cubes from the bank of unifix cubes. After they have each taken their cubes from the bank, have them put 5 cubes on the ones side at the top of their Tens and Ones charts, like this:

Tens	Ones
	❐❐❐❐❐

Next, tell the children to group their remaining cubes into tens. Ask "How many groups of ten do each of you have?" They will tell you "two" (two groups of ten). Tell them to put their two groups of ten under the word "Tens" on the left side of the chart, like this[11]:

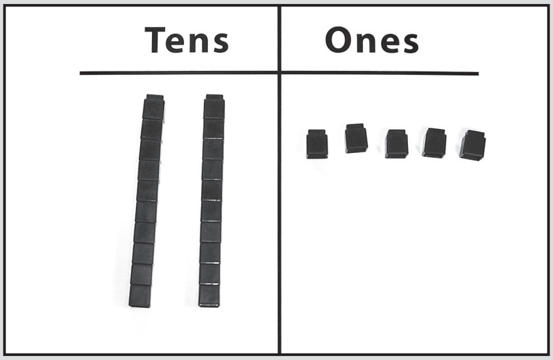

Photo 5.6

Twenty-Five Cubes on the T-Chart

Now, tell the children to take 32 cubes from their bank. After each child has done this, ask them how many groups of ten they can make with their 32 cubes. After they have formed three groups of tens, have them place them under the two groups of ten on the "tens" side of their charts. Next, ask them how many individual cubes are left now that they have used 30 to make their 3 groups of ten (two

(continued)

individual cubes are left). Have them put the two individual cubes on the ones side of their charts under the 5 ones that are already there, like this:

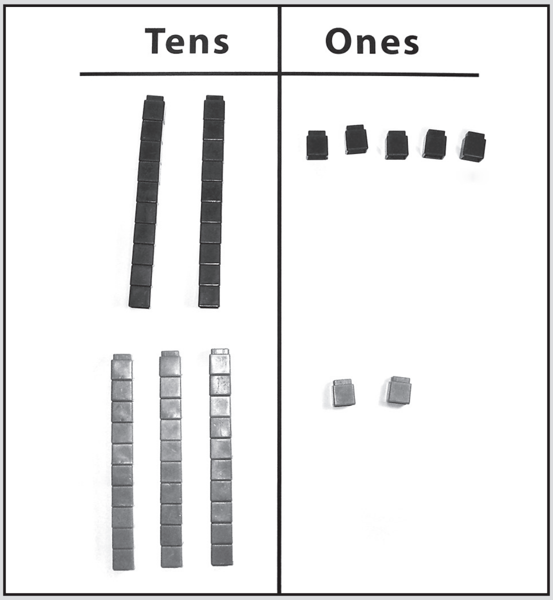

Photo 5.7
Twenty-Five Plus Thirty-Two

Next, tell your children to put together all the cubes on the "ones" side of the chart. Ask them how many cubes they have altogether on the ones side (7). Last, have them put together all of the tens on the "tens" side of their charts. Ask them how many tens they have altogether (5). Say, "So you have 5 groups of ten and 7 ones. That's 57." It should look something like this on the t-chart.

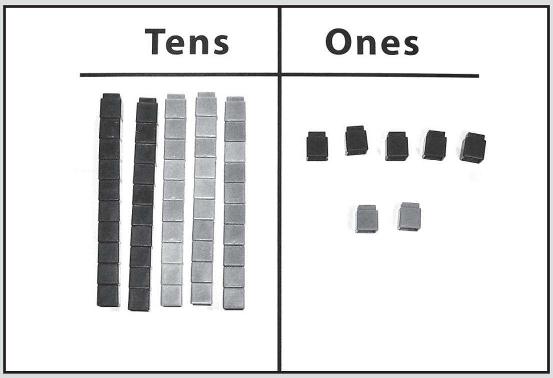

Photo 5.8
Twenty-Five Plus 32 Equals Fifty-Seven

Have the children rehearse this type of activity, using different amounts each time, until it become second nature to them. At some point, introduce the representation of the symbols. For example, write a 25 on the chalkboard and say "Set out this much [pointing to the written number, 25, on the chalkboard] on your Tens and Ones charts." After they have done this, write the number 32 under the 25 on the chalkboard and tell them to set out that much underneath the 25 cubes that are already there on their Tens and Ones charts. Next, say "Do you remember the plus sign? What does it mean?" (They should tell you that it means to put the objects together). Say "Put all the cubes together. Put all the ones together and put all the tens together. How many ones do you have? [Write the numeral 7 at the bottom of the ones place on your chalkboard]. How many tens do you have? [Write the numeral 5 at the bottom of the tens place on your chalkboard].

Teaching Children How to Add When Trading[12] Is Required: Of course, not all 2-digit addition problems are of the type for which trading ("carrying") is not required. Many 2-digit problems will require it. Children should be familiar with the process of trading ones for tens and tens for ones, as they do in the "trading game," before they are introduced to the process of 2-digit addition *with* trading. It's essential that children understand the idea that one ten is equivalent to ten ones when they learn how to solve 2-digit addition problems requiring trading.

Activity for Children 5.11

Begin this activity as you began the last one; that is, write the number 25 on a board and have the children arrange 25 cubes on their tens and ones t-charts. Next, write the number 19 underneath the 25 and have the children arrange 19 cubes on their t-charts underneath the models of 25 that they have already placed on their t-charts. At this point, their t-charts should look something like this:

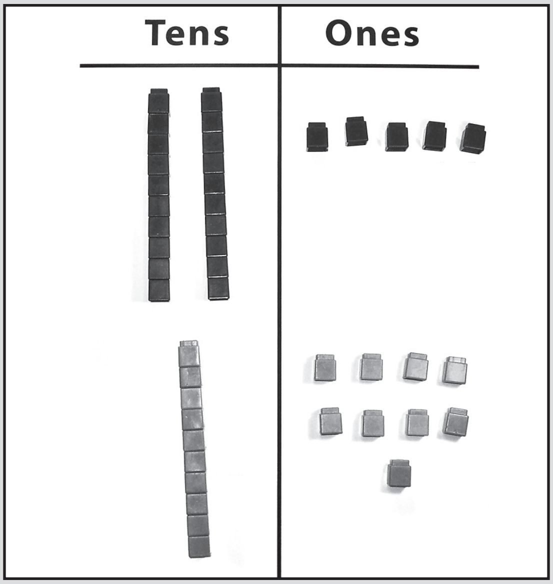

Photo 5.9
Twenty-Five Plus Nineteen

Next, remind the children that the "plus" sign means to put the two groups together (the group on the top with the group on the bottom)—ones with ones and tens with tens. Tell them to put together all of the ones cubes on the ones side of their t-charts. Now, their charts should look something like this:

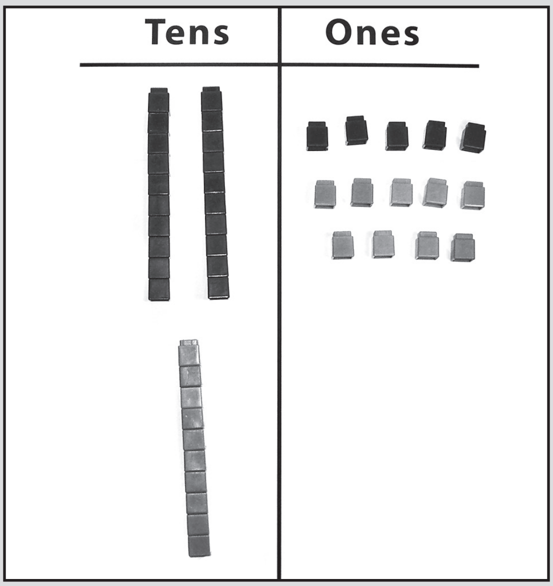

Photo 5.10
Combining the Ones in 25 + 19

Ask the children how many ones they have ("14" should be their answer) and how many tens they have in the tens side of their t-charts (3). Now, explain that we can only write one digit in the ones place, so we must make a trade of ten (of the 14 ones) for a ten and we must move the resulting ten to the tens side of the t-chart, like this:

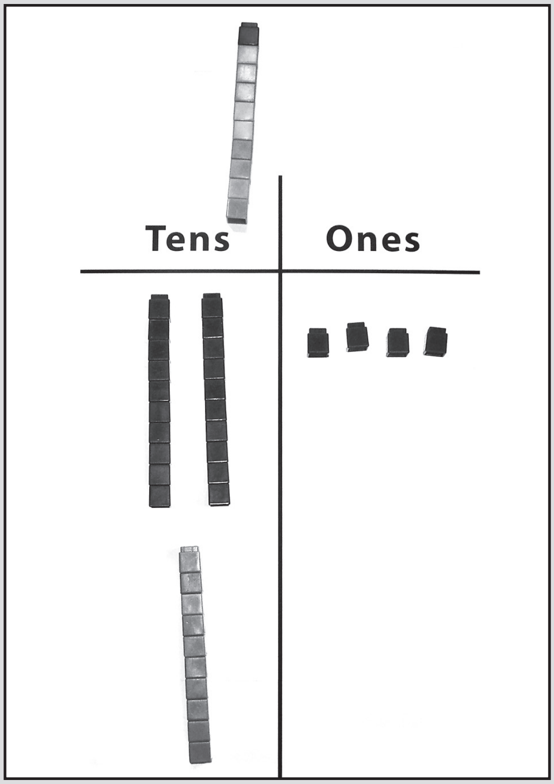

Photo 5.11
Representing the Trade on the Tens Side of the T-Chart

Notice that we put the ten that resulted from the trade *above* the other tens. Show the children that we can write symbols for these actions like this.

$$\begin{array}{r} 1 \\ 25 \\ +\,19 \\ \hline \end{array}$$

Next, ask the children how many tens are on the tens side of their t-charts (four). And then ask them how many ones are left on the ones side of their charts (four). Summarize for them by saying "So, we have four tens and four ones. That's 44!" Rehearse this type of activity with your children, providing many varied examples of 2-digit addition problems that require trading.

Teaching Children How to Subtract from 2-digit Numbers: Just as children need to learn how to add two-digit numbers, they also must learn how to subtract from 2-digit numbers. As with 2-digit addition, this skill is typically introduced in first or second grade. It is usually recommended that children be introduced to two-digit subtraction problems that don't require trading (This used to be called "borrowing"). Thus, problems such as these may be presented to the children:

$$\begin{array}{rrrr} 28 & \quad 47 & \quad 39 & \quad 53 \\ \underline{-\ \ 7} & \quad \underline{-23} & \quad \underline{-24} & \quad \underline{-31} \end{array}$$

However, problems like the following would not be presented because they require trading.

$$\begin{array}{rrrr} 28 & \quad 47 & \quad 34 & \quad 53 \\ \underline{-\ \ 9} & \quad \underline{-28} & \quad \underline{-29} & \quad \underline{-35} \end{array}$$

Activity for Children 5.12

Tell the children to arrange 28 cubes on their t-charts, like this:

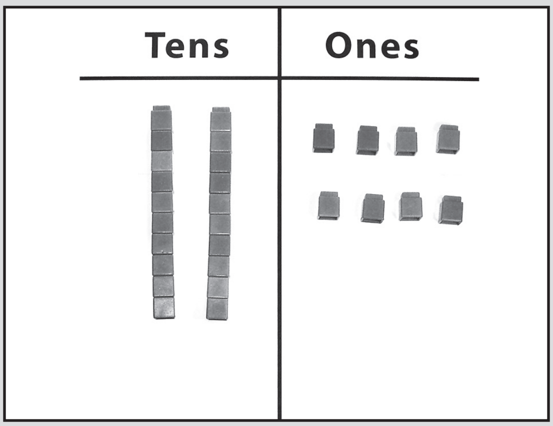

Photo 5.12

Twenty-Eight, in Tens and Ones

Write the following problem on your chalkboard.

$$\begin{array}{r} 28 \\ \underline{-13} \end{array}$$

Explain that we always subtract from the ones column before subtracting from the tens. Now, tell the children to take 13 away from the 28, taking ones from the ones column and tens from the tens column. When they have finished doing this, there charts should something like this:

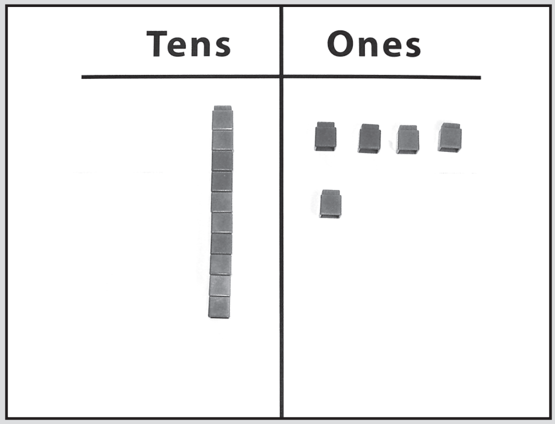

Photo 5.13

The Result of 28 – 13 (15)

Ask the children "How many ones do you have left?" ("Five") and "How many tens to you have left?" ("One"). Ask them what number has one ten and five ones ("15"). Write 15 in the row where the answer to your problem on the board is supposed to be written.

Teaching Children How to Subtract When Trading Is Required: After children have become comfortable and experienced with 2-digit subtraction that doesn't require trading, it's time to introduce them to subtraction problems that do require trading. Each of the following problems require trading because there is a larger number being subtracted (subtrahend) from the ones column than the number in the ones column at the top of the problem (minuend):

$$\begin{array}{rrr} 25 & \quad 47 & \quad 72 \\ \underline{-17} & \quad \underline{-29} & \quad \underline{-55} \end{array}$$

Activity for Children 5.13

Tell the children to arrange 34 cubes on their t-charts (Tens and Ones Charts), like this:

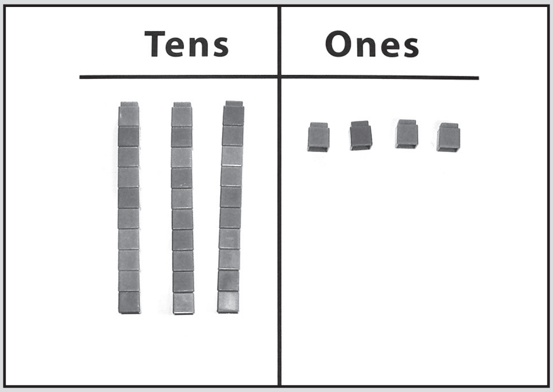

Photo 5.14
Thirty-Four in Tens and Ones

Next, tell them to subtract 18 cubes from the 34 cubes that they have laid out on their charts. Tell them to remember to start with the ones column. Say "Can you subtract 8 from 4?" (Some will try, but others will conclude that it can't get done. Discuss why it can't be done before moving on). Next, say "But you could trade one of the tens for ten ones. That would give you fourteen ones and then you could subtract 8 from 14!" Now, their t-charts should look like this:

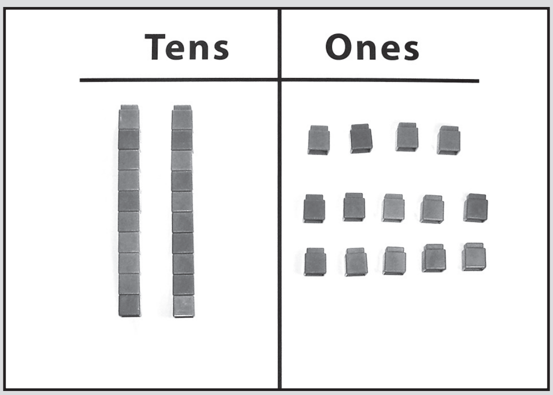

Photo 5.15
Result of Trade of One Ten for Ten Ones Combined With Four Ones

You can show this trading action with your written symbols in the following way.

$$\begin{array}{r} 34 \\ \underline{-18} \end{array} \qquad \begin{array}{r} 2\ 14 \\ \cancel{34} \\ \underline{-18} \end{array}$$

Explain that we cross out the 34 and write 2 above the tens column and 14 above the ones column to show that we traded one ten (in the tens column) for ten ones and moved them to the ones column. Now, tell the children to subtract 8 cubes from the 14 cubes in the ones column. Ask them how many ones they have left (6). Next, have the children subtract one ten from the two tens in the tens column. Ask them how many tens they have left (one). The written representation for this problem should look something like this.

$$\begin{array}{r} 2\ 14 \\ \cancel{34} \\ \underline{-18} \\ 16 \end{array}$$

There are actually several approaches to representing the trade in 2-digit subtraction problems that require trading, of course. Some people cross out the numeral in the tens column and then insert a one just to the left of the numeral in the ones column (a 4, in this case). To reduce confusion, we recommend teaching the children to represent the trading process in whatever format their math workbooks use.

A Word About Using Manipulatives for 2-Digit Addition and Subtraction: Many teachers make use of manipulatives and trading actions when introducing a concept such as addition with trading but they don't bother to continue doing so. This is almost as bad as if the children had never been introduced to trading at all! Children need continuous experiences with manipulatives (and trading ones for tens) if they are to develop an understanding of the concept being taught. Our philosophy is that children should be encouraged to use manipulatives to solve problems until they no longer seem to need to do so. But how can a teacher judge when a child no longer needs them? We typically make the manipulatives available and leave it to the child to decide when they no longer need them. If a child *asks* "Do I *have* to use the cubes?", we say "No." If we notice that a particular child's reasoning about number operations shows that she or he no longer needs to use manipulatives, we encourage the child to solve problems without using them. Likewise, if we notice that a child is not using manipulatives but seems not to have acquired a basic understanding of the trading action as it applies to 2-digit addition or subtraction, we sit down with the child and model the process with manipulatives.

Learning to Divide: There are two types of division and, like subtraction, children must learn how to solve more than one type of division problem in order to be successful in applying arithmetic to their everyday lives.

Partitive Division ("One for me, one for you"). Children naturally model partitive division problems before they know what "division" means. For example, you might give 9 caramels to 3 children and tell them to share them equally. They will quickly figure out how many each child should get. One way that they might solve this problem is to say "One for me, one for you, and one for you" and repeat this process three times until all the caramels have been given to someone.

Measurement Division (repeated subtraction). Another way to think of division (another type of division problem) is usually referred to as "measurement division." This is the idea that we can repeatedly subtract the same amount from the dividend until we have nothing left to subtract. For example, for $12 \div 4$, we can say $12 - 4 = 8$ and $8 - 4 = 4$ and $4 - 4 = 0$. Now, we must ask ourselves "How many times did we subtract 4." The answer is 3.

Activity for Children 5.14

Have children work in pairs. Give each pair 4 stickers and tell them to share them equally. When they have done so, summarize for the children by saying "So, you divided 4 stickers up equally between two persons. How many stickers did each of you get?" (two). Show them the symbolic representation for this problem: $4 \div 2 = 2$. Repeat this activity with other amounts; for example, $6 \div 2 = 3$, $8 \div 2 = 4$, and so on. Next, group children in 3s and have them model more partitive problems; for example, $9 \div 3 = 3$, $12 \div 3 = 4$, etc. Next, group children in 4s and have them model more problems; for example, $12 \div 4 = 3$, $16 \div 4 = 4$, and so on. Avoid problems with remainders for the meanwhile, until they have become fairly skilled with this type of problem.

Activity for Children 5.15

We can model measurement division with objects in the following way. First, tell the children to set out 12 cubes on their desks or charts, like this:

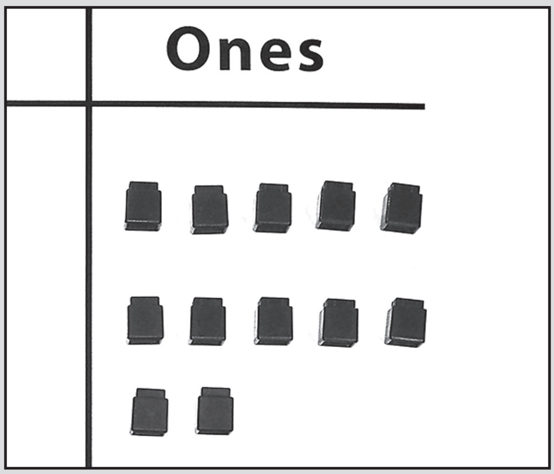

Photo 5.16

Starting With 12 Cubes (Measurement Division)

Next, tell the children to remove 4 cubes. Thus, starting with 12 cubes, the child removes 4, leaving 8 cubes on the chart, as in the photo below.

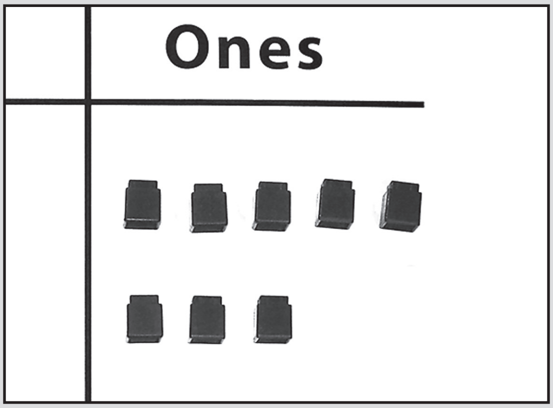

Photo 5.17

Result of 12 – 4 (Measurement Division)

Next, the child removes 4 more cubes, leaving 4 on her chart, as pictured below.

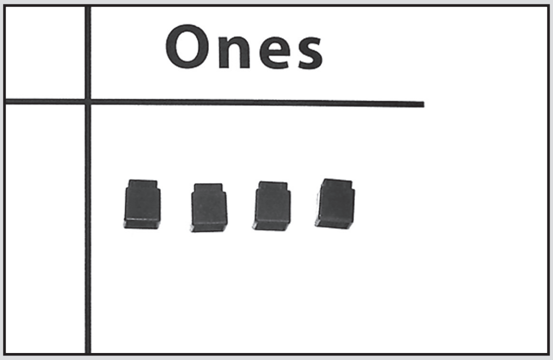

Photo 5.18

Result of 8 – 4 (Measurement Division)

Finally, the child removes 4 more cubes, leaving no cubes at all.

Now, ask the children to figure out how many times they removed 4 from the original 12. Their answer should be 3. When they tell you this, write this equation on a board or slate: 12 ÷ 4 = 3. Summarize for the children "So, we took away 4 cubes at a time. We started with 12 and we ended up with none. Zero! How many fours did we take away from the twelve we started with? How many times did we take away four cubes from the 12 cubes we began with?" (They should say "Three").

STANDARDS

Understand meanings of operations and how they relate to one another

Pre-K–2 Expectations: In pre-K through grade 2 all students should:

- understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations;
- understand the effects of adding and subtracting whole numbers;
- understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally.

Source: NCTM Standards for Numbers and Number Operations

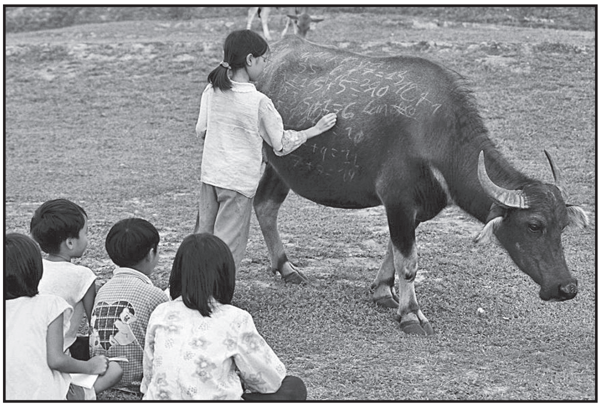

Photo 5.19

Math Lesson in Vietnam

Glossary

digit: a single numeral of 0–9 that occupies a single place in a numeral; for example, the "ones place," "tens place," or "hundreds place."

number: an amount.

numeral: the symbols for an amount.

Endnotes

[1] In mathematics, "groups" is actually a term for something else.

[2] Acorns can be sprayed with a fixative so they won't decompose.

[3] The equals sign doesn't mean this at all, but research by Kamii (1985, 2000) and others has shown that young children don't really understand the concept of equations. In order to familiarize young children with the process of addition and the symbols that represent it, we have used this approach to teaching them about addition. Telling children that the equals sign means "is the same as" is not much better than saying "equals" since it's the child's inability to understand class inclusion at this stage that interferes with the comprehension of the concept that both sides of the equation are the same. Later (for example, in grade 4), children can be introduced to the real meaning of equations.

[4] This is not technically correct, in mathematical terms. However, if children are to model solving equations during the early grades, they will need an explanation that matches the procedure they use to model these equations.

[5] This game was adapted and significantly changed from C. Kamii's description of *Piggy Bank* (Kamii, 2000, p. 181).

[6] This may not be technically correct in terms of mathematics, but it helps the child to understand the process of modeling the action of multiplication.

[7] Similar tracks are sold with other brands of interlocking cubes designed for use in teaching children about number. Number tracks serve several purposes. We have used them to teach children about counting objects, about addition or subtraction, and about skip-counting.

[8] You don't need to use an actual board to play this game. You can use a table or a space on the floor just as easily.

[9] Note: With young children, we recommend using dice with dots on them, as opposed to numerals, until the child's number knowledge is secure. However, children who have reached the stage where they can learn about multiplication can use either type of die.

[10] Any manipulative that can be combined into groups of tens may be used (e.g., Base ten blocks, unifix cubes, popsicle sticks, etc.).

[11] Note: These manipulatives are unifix cubes. However, any manipulatives that can be grouped in tens may be used; for example, popsicle sticks may be grouped in rubber bands. Base ten blocks come with ones blocks and tens blocks (The "ones blocks" are 1 cm long while the "tens blocks" are 10 cm long. The tens blocks are scored to show where each individual block ends and begins, creating the impression that 10 ones blocks have been glued together).

[12] Names other than "trading" have been used to refer to this process; for example, "carrying," "renaming", and "regrouping." All of these names *except* "carrying" are appropriate. The essential idea that children must grasp is that ones are being exchanged for tens whenever the amount of ones is greater than 10. "Carrying" is not appropriate because it doesn't connote this exchange. Instead, it implies a magical transformation that defies explanation. This is undesirable because math must make sense to children if they are to understand it.

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About the Authors

Robert Louisell holds a doctorate in elementary and early childhood education from University of Illinois and is Professor Emeritus at St. Cloud State University. He has seven years of teaching experience in public schools at the early childhood level. He has taught in early childhood education programs at St. Ambrose University and University of Texas, El Paso, and in the elementary education program at St. Cloud State University. He is also the co-author of *Developing a Teaching Style* (Harper-Collins, 1992, Waveland Press, 2001).

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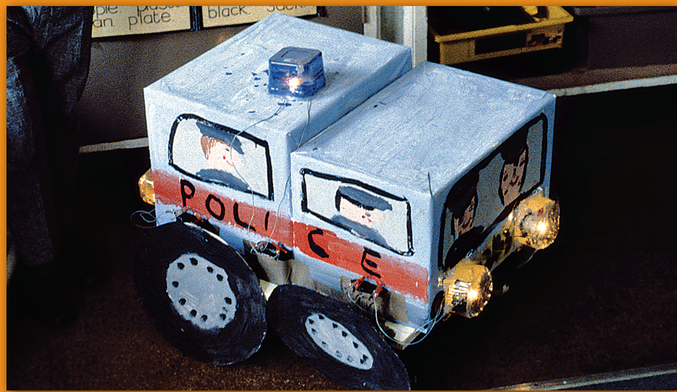
I DO, AND I UNDERSTAND

Helping Young Children Discover Science and Mathematics

"A merit of his liberalized Piagetian approach is to help educators to take a subject's perspective about children as learners. These analyses of Dr. Louisell's yield wise advice on educational strategies, which might help to refine interventions in elementary science education."
— Juan Pascual-Leone

(About science activities)
"If we provide hands-on situations for children—for example, exploring density by testing out which objects sink and float or learning about electricity by experimenting with batteries, bulbs, and wires—they will reflect on the science phenomena that they observe and develop their own ideas about them."



Facsimile of a Police Car Built By Children

Paintings by 1st Grade Children After Trip to Zoo

(About constructivism)
"When we help children to develop their *own* ideas about the world around them... we are helping them develop *their* knowledge of the world. This is far more important than trying to pass on our own knowledge — a knowledge most young children will be unable to understand at this stage."

"Previous texts on the subject … neglected … grade levels one through three. This book … comprehensively deals with preschool/kindergarten *and* grades 1–3 and it contains over a hundred tested early childhood activities for mathematics and science."

Robert Louisell holds a doctorate in elementary and early childhood education from University of Illinois and is Professor Emeritus at St. Cloud State University. He has seven years of teaching experience in public schools at the early childhood level. He has taught in early childhood education programs at St. Ambrose University and University of Texas, El Paso, and in the elementary education program at St. Cloud State University. He is also the co-author of *Developing a Teaching Style* (Harper-Collins, 1992, Waveland Press, 2001).

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